IB WORLD SCHOOL 1309 (ZSO13 Gdańsk)

Mathematics: analysis and approaches

syllabus & course of study

(based on Mathematics: analysis and approaches

guide,

first exams 2021,

SL: 5h/week in the first year and 6h/week in the second year

HL: 7h/week in the first year and 9h/week in the second year)

A. COURSE AIMS

The aims of all DP mathematics courses are to enable students to:

- develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
- develop an understanding of the concepts, principles and nature of mathematics
- communicate mathematics clearly, concisely and confidently in a variety of contexts
- develop logical and creative thinking, and patience and persistence in problem solving to instil confidence in using mathematics
- employ and refine their powers of abstraction and generalization
- take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities
- appreciate how developments in technology and mathematics influence each other
- appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics
- appreciate the universality of mathematics and its multicultural, inter-national and historical perspectives
- appreciate the contribution of mathematics to other disciplines, and as a particular "area of knowledge" in the TOK course
- develop the ability to reflect critically upon their own work and the work of others
- independently and collaboratively extend their understanding of mathematics.

B. COURSE OBJECTIVES

Problem-solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems.

The assessment objectives are common to Mathematics: analysis and approaches and to Mathematics: applications and interpretation.

- Knowledge and understanding: Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.
- Problem solving: Recall, select and use their knowledge of mathematical skills, results and models in both abstract and real-world contexts to solve problems.
- Communication and interpretation: Transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation; use appropriate notation and terminology.
- Technology: Use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems.
- Reasoning: Construct mathematical arguments through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions.
- Inquiry approaches: Investigate unfamiliar situations, both abstract and from the real world, involving organizing and analysing information, making conjectures, drawing conclusions, and testing their validity.

The exploration is an integral part of the course and its assessment, and is compulsory for both SL and HL students. It enables students to demonstrate the application of their skills and knowledge, and to pursue their personal interests, without the time limitations and other constraints that are associated with written examinations.

C. COURSE OVERVIEW

C1. Course Content:

C.1.Syllabus component:

- number and algebra
- functions
- geometry and trigonometry
- statistics and probability
- calculus
- development of investigational, problem-solving and modelling skills and the exploration of an area of mathematics

C.2.Textbook& reference books

SL: I. Wazir, T. Garry *Mathematics: Analysis and Approaches Standard Level for the IB Diploma*
HL: I. Wazir, T. Garry *Mathematics: Analysis and Approaches Higher Level for the IB Diploma*

C.3 Other requirements

TI-Nspire CX Calculator

Other issues are settled in relevant school documents- academic integrity, assessment, inclusion and language policies

D. COURSE OF STUDY

THEMES/ UNITS	**Content- topics**		Time provision
	Number and algebra	SL/HL • Operations with numbers in the form a × 10k where 1 ≤ a < 10 and k is an integer. • Arithmetic sequences and series. Use of the formulae for the n th term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences. • Geometric sequences and series. Use of the formulae for the n th term and the sum of the first n terms of the sequence. • Use of sigma notation for the sums of geometric sequences. • Financial applications of geometric sequences and series: compound interest and annual depreciation. • Laws of exponents with integer exponents. • Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology. • Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side (LHS to RHS) proof. The symbols and notation for equality and identity. • Laws of exponents with rational exponents. • Laws of logarithms. $\log_a xy = \log_a x + \log_a y$ $\log_a x/y = \log_a x - \log_a y$ $\log_a x^m = m\log_a x$ for a, x, y > 0 • Change of base of a logarithm.	30 hours

		• Solving exponential equations, including using logarithms. • Sum of infinite convergent geometric sequences • The binomial theorem • Use of Pascal's triangle and $^{n}C_{r}$. HL only • Counting principles, including permutations and combinations. • Extension of the binomial theorem to fractional and negative indices, ie $(a + b)^n$, $n \in \mathbb{Q}$. • Partial fractions. • Complex numbers: the number i, where i 2 = − 1. Cartesian form z = a + bi; the terms real part, imaginary part, conjugate, modulus and argument. • The complex plane. • Modulus–argument (polar) form: $z = r(\cos\theta + i\sin\theta) = r\text{cis}\theta$. Euler form: $z = re^{i\theta}$ • Sums, products and quotients in Cartesian, polar or Euler forms and their geometric interpretation. • Complex conjugate roots of quadratic and polynomial equations with real coefficients. • De Moivre's theorem and its extension to rational exponents. • Powers and roots of complex numbers. • Proof by mathematical induction. • Proof by contradiction • Use of a counterexample to show that a statement is not always true. • Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution.	20 hours
	Functions	SL/HL • Different forms of the equation of a straight line. Gradient; intercepts. Lines with gradients m_1 and m_2 Parallel lines $m_1 = m_2$. Perpendicular lines $m_1 \times m_2 = -1$ • Concept of a function, domain, range and graph. Function notation, for example f(x), v(t), C(n). The concept of a function as a mathematical model. • Informal concept that an inverse function reverses or undoes the effect of a function.	60 hours

		Inverse function as a reflection in the line y = x, and the notation f^{-1} • The graph of a function; its equation • Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences. • Determine key features of graphs. • Finding the point of intersection of two curves or lines using technology. • Composite functions • Identity function. Finding the inverse function. • The quadratic function f(x) = ax2 + bx + c: its graph, y -intercept (0, c). Axis of symmetry. The form f(x) = a(x − p)(x − q), xintercepts (p, 0) and (q, 0). The form $f(x) = a(x-h)^2 + k$, vertex (h, k) • Solution of quadratic equations and inequalities. The quadratic formula. • The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots. • The reciprocal function f(x) = 1 x , x ≠ 0: its graph and self-inverse nature. • Rational functions of the form f(x) = (ax + b)/ (cx + d) and their graphs. Equations of vertical and horizontal asymptotes. • Exponential functions and their graphs. • Logarithmic functions and their graphs. • Solving equations, both graphically and analytically • Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. • Applications of graphing skills and solving equations that relate to real-life situations. • Transformations of graphs. Translations: y = f(x) + b; y = f(x − a). Reflections (in both axes): y = − f(x); y = f(− x). Vertical stretch with scale factor p: y = p f(x). Horizontal stretch with scale factor 1/q : y = f(qx) • Composite transformations HL only • Polynomial functions, their graphs and equations; zeros, roots and factors. The factor and remainder theorems.	20 hours

		• Sum and product of the roots of polynomial equations • Rational functions • Odd and even functions. • Finding the inverse function, $f^{-1}(x)$, including domain restriction. • Self-inverse functions • Solutions of $g(x) \geq f(x)$, both graphically and analytically. • The graphs of the functions, $y = \| f(x)\|$ and $y = f(\|x\|)$, $y = \frac{1}{f(x)}$, $y = f(ax+b)$, $y = [f(x)]^2$.	
	Geometry and trigonometry	SL/HL • The distance between two points in threedimensional space, and their midpoint • Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane. • Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$. Area of a triangle as $\frac{1}{2}ab\sin C$. • Applications of right and non-right angled trigonometry, including Pythagoras's theorem. Angles of elevation and depression. • Construction of labelled diagrams from written statements. • The circle: radian measure of angles; length of an arc; area of a sector. • Definition of $\cos\theta$, $\sin\theta$ in terms of the unit circle. • Definition of $\tan\theta$ as $\sin\theta / \cos\theta$. • Exact values of trigonometric ratios of 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$ and their multiples. • Extension of the sine rule to the ambiguous case. • The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$. Double angle identities for sine and cosine.	60 hours

		• The relationship between trigonometric ratios. • The circular functions sinx, cosx, and tanx; • amplitude, their periodic nature, and their graphs Composite functions of the form f(x) = asin(b(x + c)) + d • Transformations. • Real-life contexts. • Solving trigonometric equations in a finite interval, both graphically and analytically. • Equations leading to quadratic equations in sinx, cosx or tanx HL only • Definition of the reciprocal trigonometric ratios secθ, cosecθ and cotθ. Pythagorean identities: $1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \text{cosec}^2 \theta$ The inverse functions f(x) = arcsinx, f(x) = arccosx, f(x) = arctanx; their domains and ranges; their graphs • Compound angle identities. Double angle identity for tan. • Relationships between trigonometric functions and the symmetry properties of their graphs. Concept of a vector; position vectors; displacement vectors. Representation of vectors using directed line segments. Base vectors $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$. Components of a vector: $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}.$ • Algebraic and geometric approaches to the following: the sum and difference of two vectors; the zero vector 0, the vector –v; multiplication by a scalar, kv, parallel vectors magnitude of a vector, |v|; unit vectors, v |v|; position vectors OA → = a, OB → = b; displacement vector AB → = b − a Proofs of geometrical properties using vectors. • The definition of the scalar product of two vectors. The angle between two vectors. Perpendicular vectors; parallel vectors • Vector equation of a line in two and three dimensions: r = a + λb. • The angle between two lines. • Simple applications to kinematics.	30 hours

		- Coincident, parallel, intersecting and skew lines, distinguishing between these cases. Points of intersection. - The definition of the vector product of two vectors. - Properties of the vector product. - Geometric interpretation of $\| v \times w \|$ - Vector equations of a plane: $r = a + \lambda b + \mu c$, where b and c are non-parallel vectors within the plane. $r \cdot n = a \cdot n$, where n is a normal to the plane and a is the position vector of a point on the plane. Cartesian equation of a plane $ax + by + cz = d$ - Intersections of: a line with a plane; two planes; three planes. Angle between: a line and a plane; two planes	
	Statistics and probability	SL/HL - This is designed to cover the key questions that students should ask when they see a data set/ analysis - Dealing with missing data, errors in the recording of data - Interpretation of outliers - Sampling techniques and their effectiveness. - Presentation of data (discrete and continuous): frequency distributions (tables). - Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR). - Production and understanding of box and whisker diagrams. - Measures of central tendency (mean, median and mode). Estimation of mean from grouped data. - Modal class. - Measures of dispersion (interquartile range, standard deviation and variance). - Effect of constant changes on the original data. - Quartiles of discrete data. - Linear correlation of bivariate data. Pearson's product-moment correlation coefficient, r. - Scatter diagrams; lines of best fit, by eye, passing through the mean point. - Equation of the regression line of y on x	60 hours

		- Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression y = ax + b - Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event. The probability of an event A is P(A) = n(A) n(U) . The complementary events A and A' (not A). - Expected number of occurrences. - Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities. - Combined events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Mutually exclusive events: $P(A \cap B) = 0$. - Conditional probability: $P(A\|B) = P(A \cap B) / P(B)$. - Independent events: $P(A \cap B) = P(A)P(B)$. - Concept of discrete random variables and their probability distributions. Expected value (mean), for discrete data. Applications. - Binomial distribution. Mean and variance of the binomial distribution. - The normal distribution and curve. Properties of the normal distribution. Diagrammatic representation. - Normal probability calculations. - Inverse normal calculations - Equation of the regression line of x on y. - Use of the equation for prediction purposes. - Formal definition and use of the formulae: $P(A\|B) = P(A \cap B)/ P(B)$ for conditional probabilities, and $P(A\|B) = P(A) = P(A\|B')$ for independent events. - Standardization of normal variables (z-values). - Inverse normal calculations where mean and standard deviation are unknown. HL only - Use of Bayes' theorem for a maximum of three events. - Variance of a discrete random variable. - Continuous random variables and their probability density functions - Mode and median of continuous random variables	20 hours

		• Mean, variance and standard deviation of both discrete and continuous random variables. • The effect of linear transformations of X.	
	Calculus	SL/HL • Introduction to the concept of a limit. • Derivative interpreted as gradient function and as rate of change. • Increasing and decreasing functions. Graphical interpretation of f ′(x) > 0, f ′(x) = 0, f ′(x) < 0. • Derivative of $f(x) = ax^n$ is $f'(x) = anx^{n-1}$, $n \in \mathbb{Z}$ The derivative of functions of the form $f(x) = ax^n + bx^{n-1} \ldots$ where all exponents are integers. • Tangents and normals at a given point, and their equations. • Introduction to integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \ldots$, where $n \in \mathbb{Z}$, $n \neq -1$ • Anti-differentiation with a boundary condition to determine the constant term • Definite integrals using technology. Area of a region enclosed by a curve y = f(x) and the x -axis, where f(x) > 0. • Derivative of x^n ($n \in \mathbb{Q}$), sinx, cosx, e^x and lnx. Differentiation of a sum and a multiple of these functions. • The chain rule for composite functions • The product and quotient rules. • The second derivative. Graphical behaviour of functions, including the relationship between the graphs of f , f ′ and f ″. • Local maximum and minimum points. Testing for maximum and minimum. • optimization. • Points of inflexion with zero and non-zero gradients. • Kinematic problems involving displacement s, velocity v, acceleration a and total distance travelled • Indefinite integral of x^n ($n \in \mathbb{Q}$), sinx, cosx, 1/ x and e^x • The composites of any of these with the linear function ax + b.	40 hours

		Integration by inspection (reverse chain rule) or by substitution for expressions of the form: $\int kg'(x)f(g(x))\mathrm{d}x$. • Definite integrals, including analytical approach. HL only • Informal understanding of continuity and differentiability of a function at a point. Understanding of limits (convergence and divergence). Definition of derivative from first principles $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. • Higher derivatives The evaluation of limits of the form $\lim_{x \to a} \frac{f(x)}{g(x)}$ and $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ using l'Hôpital's rule or the Maclaurin series. Repeated use of l'Hôpital's rule. • Implicit differentiation. Related rates of change. Optimisation problems. • Derivatives of tanx, secx, cosecx, cotx, a^x , $\log_a x$, arcsinx, arccosx, arctanx. • Indefinite integrals of the derivatives of any of the above functions. The composites of any of these with a linear function. • Use of partial fractions to rearrange the integrand. • Integration by substitution. • Integration by parts • Repeated integration by parts • Area of the region enclosed by a curve and the yaxis in a given interval. Volumes of revolution about the x-axis or y-axis • First order differential equations. Numerical solution of dy / dx = f(x, y) using Euler's method. • Variables separable • Homogeneous differential equation dy /dx = f(y/ x) using the substitution y = vx. • Solution of y' + P(x)y = Q(x), using the integrating factor	30 hours	

			Maclaurin series to obtain expansions for e^x, $\sin x$, $\cos x$, $\ln(1+x)$, $(1+x)^p$, $p \in \mathbb{Q}$. Use of simple substitution, products, integration and differentiation to obtain other series. Maclaurin series developed from differential equations.	
Exploration	Knowledge and understanding Problem solving Communication and interpretation Technology Reasoning Inquiry approaches			25 hours
Exam skills	Solving problems from specimen/past papers and exam-style questions Writing mock exams Use of technology			25 hours
Knowledge (revised and/or constructed)	Knowledge relating to the 5 prescribed themes Use of mathematical language and notation Exam strategies and techniques **EE ideas, TOK and CAS links explored throughout all the course themes**			
Concepts	Approximation Change Equivalence Generalization Modelling Patterns Quantity Relationships Representation Space Systems Validity			
Skills (developed and practised)	**Language skills:** Productive skills: writing, speaking Receptive skills: listening and reading comprehension Interactive skills: speaking ✓ interpersonal skills ✓ reasoning ✓ research ✓ groupwork ✓ creative thinking ✓ organization			

	ATL (approaches to learning) skills: self-management skills time management—including using time effectively in class, keeping to deadlines, keeping to class schedules, creating study planners, homework planners and sticking to them self-management—including personal goal setting, organization of learning materials, turning up to class with the right materials organizing information logically, structuring information correctly in essays, and reports using different information organizers for different purposes demonstrating persistence and perseverance, practicing focus and concentration, and overcoming distraction social skills: working in groups—including delegating and taking responsibility, adapting to roles, resolving group conflicts, demonstrating teamwork accepting others—including analysing others' ideas, respecting others' points of view, and using ideas critically personal challenges—including respecting cultural differences, negotiating goals and limitations with peers and with teachers, taking responsibility for own actions communication skills: Active listening----including non-verbal communication, and listening to and following directions and procedures literacy—including reading strategies, using and interpreting a range of contentspecific terminology, interpreting meaning through cultural understanding being informed—including the use of a variety of media, receiving feedback informing others—including presentation skills using a variety of media, clear speaking, writing for different purposes and giving feedback research skills: accessing information—including researching from a variety of sources, transferring and summarizing information using a range of technologies, identifying primary and secondary sources selecting and organizing information— including identifying points of view,bias and weaknesses, using primary and secondary sources, making connections between a variety of resources, collecting, recording and verifying data referencing—including the use of citing, footnotes and referencing of sources, respecting the concept of intellectual property rights thinking skills: generating ideas—including the use of brainstorming planning—including storyboarding and outlining a plan inquiring—including questioning and challenging information and arguments, developing questions, developing the skills of critical analysis and using the inquiry cycle applying knowledge and concepts—including logical progression of arguments identifying problems—including deductive reasoning, evaluating solutions to problems creating novel solutions—including the combination of critical and creative strategies, considering a problem from multiple perspectives

	self-awareness—including seeking out positive criticism, reflecting on areas of perceived limitation self-evaluation—including the keeping of learning journals reflecting at different stages in the learning process on learning experiences in order to support personal development through thinking about meta-cognition (the way we think and learn) making connections—including using knowledge, understanding and skills across subjects to create products or solutions, applying skills and knowledge in unfamiliar situations inquiring in different contexts— including changing the context of an inquiry to gain various perspectives.
Attitudes (encouraged and fostered)	**Inquirers** They develop their natural curiosity. They acquire the skills necessary to conduct inquiry and research and show independence in learning. They actively enjoy earning and this love of learning will be sustained throughout their lives. **Knowledgeable** They explore concepts, ideas and issues that have local and global significance. In so doing, they acquire in-depth knowledge and develop understanding across a broad and balanced range of disciplines. **Thinkers** They exercise initiative in applying thinking skills critically and creatively to analyze and take responsible action on complex problems, and make reasoned, ethical decisions. **Communicators** They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication. They work effectively and willingly in collaboration with others. **Principled** They act with integrity and honesty, with a strong sense of fairness, justice and respect for the dignity of the individual, groups and communities. They take responsibility for their own actions and the consequences that accompany them. **Open-minded** They understand and appreciate their own cultures and personal histories, and are open to the perspectives, values and traditions of other individuals and communities. They are accustomed to seeking and evaluating a range of points of view, and are willing to grow from the experience. **Caring** They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to service, and act to make a positive difference to the lives of others and to the environment. **Risk-takers** They approach unfamiliar situations and uncertainty with courage and forethought, and have the independence of spirit to explore new roles, ideas and innovative strategies. They are resourceful and resilient in the face of challenges and change. **Balanced** They understand the importance of intellectual, physical and emotional balance to achieve personal well-being for themselves and others. They recognize their interdependence with other people and with the world in which they live. **Reflective** They give thoughtful consideration to their own learning and experience. They are able to assess and understand their strengths and limitations in order to support their learning and personal development.

(More detailed content and specific skills, knowledge, concepts build and/or explored in unit planners)

E. ASSESSMENT

E.1 Assessment at a glance

Type of assessment	Format of assessment	Time (hours)		Weighting of final grade (%)	
		SL	HL	SL	HL
External					
Paper 1	No technology allowed. Section A: compulsory short-response questions based on the syllabus. Section B: compulsory extended-response questions based on the syllabus.	1.5	2	40	30
Paper 2	Technology allowed. Section A: compulsory short-response questions based on the syllabus. Section B: compulsory extended-response questions based on the syllabus.	1.5	2	40	30
Paper 3	Technology allowed. Two compulsory extended-response problem-solving questions		1		20
Internal					
Exploration		15	15	20	20

E.2 Assessment criteria

General

Mark schemes are used to assess students in all papers. The mark schemes are specific to each examination.

External assessment details—SL

General information

Paper 1 and paper 2

These papers are externally set and externally marked. Together, they contribute 80% of the final mark for the course. These papers are designed to allow students to demonstrate what they know and what they can do.

Papers 1 and 2 will contain some questions, or parts of questions, which are common with HL.

Calculators

Paper 1

Students are not permitted access to any calculator. Questions will mainly involve analytic approaches to solutions, rather than requiring the use of a GDC. The paper is not intended to require complicated calculations, with the potential for careless errors. However, questions will include some arithmetical manipulations when they are essential to the development of the question.

Paper 2

Students must have access to a graphic display calculator (GDC) at all times. However, not all questions will necessarily require the use of the GDC. Regulations covering the types of GDC allowed are provided in Diploma Programme *Assessment procedures*.

Formula booklet

Each student must have access to a clean copy of the formula booklet during the examination. It is the responsibility of the school to download a copy from IBIS or the programme resource centre and to ensure that there are sufficient copies available for all students.

Awarding of marks

Marks are awarded for method, accuracy, answers and reasoning, including interpretation.

In paper 1 and paper 2, full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations (in the form of, for example diagrams, graphs or calculations). Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. All students should therefore be advised to show their working.

Paper 1

Duration: 1 hour 30 minutes

Weighting: 40%

- This paper consists of section A, short-response questions, and section B, extended-response questions.

- Students are not permitted access to any calculator on this paper.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.

Mark allocation

- This paper is worth **80** marks, representing **40%** of the final mark.
- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily each be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

Section A

- This section consists of compulsory short-response questions based on the whole syllabus. It is worth approximately 40 marks.
- The intention of this section is to assess students across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- A small number of steps are needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

Section B

- This section consists of a small number of compulsory extended-response questions based on the whole syllabus. It is worth approximately 40 marks.
- Individual questions may require knowledge of more than one topic.
- The intention of this section is to assess students across the breadth of the syllabus in depth. The range of syllabus topics tested in this section may be narrower than that tested in section A.

Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions will develop a single theme.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on sustained reasoning.

Paper 2

Duration: 1 hour 30 minutes

Weighting: 40%

- This paper consists of section A, short-response questions, and section B, extended-response questions.
- A GDC is required for this paper, but not every question will necessarily require its use.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.

Mark allocation

- This paper is worth **80** marks, representing **40%** of the final mark.

- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily each be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

Section A

- This section consists of compulsory short-response questions based on the whole syllabus. It is worth approximately 40 marks.
- The intention of this section is to assess students across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- A small number of steps are needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

Section B

- This section consists of a small number of compulsory extended-response questions based on the whole syllabus. It is worth approximately 40 marks.
- Individual questions may require knowledge of more than one topic.
- The intention of this section is to assess students across the breadth of the syllabus in depth. The range of syllabus topics tested in this section may be narrower than that tested in section A.

Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions will develop a single theme.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on sustained reasoning.

General

Markschemes are used to assess students in all papers. The markschemes are specific to each examination.

External assessment details—HL

General information

Papers 1, 2 and 3

These papers are externally set and externally marked. Together, they contribute 80% of the final mark for the course. These papers are designed to allow students to demonstrate what they know and what they can do.

Papers 1 and 2 will contain some questions, or parts of questions, which are common with SL.

Calculators

Paper 1

Students are not permitted access to any calculator. Questions will mainly involve analytic approaches to solutions, rather than requiring the use of a GDC. The paper is not intended to require complicated calculations, with the potential for careless errors. However, questions will include some arithmetical manipulations when they are essential to the development of the question.

Paper 2

Students must have access to a GDC at all times. However, not all questions will necessarily require the use of the GDC. Regulations covering the types of GDC allowed are provided in *Diploma Programme Assessment procedures*.

Paper 3

Students must have access to a GDC at all times. However, not all question parts will necessarily require the use of the GDC. Regulations covering the types of GDC allowed are provided in Diploma Programme *Assessment procedures*.

Formula booklet

Each student must have access to a clean copy of the formula booklet during the examination. It is the responsibility of the school to download a copy from IBIS or the Programme Resource Centre and to ensure that there are sufficient copies available for all students.

Awarding of marks

Marks are awarded for method, accuracy, answers and reasoning, including interpretation.

In papers 1, 2 and 3, full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations (in the form of, for example diagrams, graphs or calculations). Where an answer is incorrect, some marks may be given for correct method, provided that this is shown by written working. All students should therefore be advised to show their working.

Paper 1

Duration: 2 hours

Weighting: 30%

- This paper consists of section A, short-response questions, and section B, extended-response questions.
- Students are not permitted access to any calculator on this paper.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.

Mark allocation

- This paper is worth **110** marks, representing **30%** of the final mark.
- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily each be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

Section A

- This section consists of compulsory short-response questions based on the whole syllabus. It is worth approximately 55 marks.
- The intention of this section is to assess students across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- A small number of steps are needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

Section B

This section consists of a small number of compulsory extended-response questions based on the whole syllabus. It is worth approximately 55 marks.

Individual questions may require knowledge of more than one topic.

The intention of this section is to assess students across the breadth of the syllabus in depth. The range of syllabus topics tested in this section may be narrower than that tested in section A.

Question type

- Questions require extended responses.
- Individual questions will develop a single theme.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on sustained reasoning.

Paper 2

Duration: 2 hours

Weighting: 30%

- This paper consists of section A, short-response questions, and section B, extended-response questions.
- A GDC is required for this paper, but not every question will necessarily require its use.

Syllabus coverage

- Knowledge of **all** topics is required for this paper. However, not all topics are necessarily assessed in every examination session.

Mark allocation

- This paper is worth **110** marks, representing **30%** of the final mark.
- Questions of varying levels of difficulty and length are set. Therefore, individual questions may not necessarily each be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of the question.

Section A

- This section consists of compulsory short-response questions based on the whole syllabus. It is worth approximately 55 marks.
- The intention of this section is to assess students across the breadth of the syllabus. However, it should not be assumed that the separate topics are given equal emphasis.

Question type

- A small number of steps are needed to solve each question.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.

Section B

- This section consists of a small number of compulsory extended-response questions based on the whole syllabus. It is worth approximately 55 marks.
- Individual questions may require knowledge of more than one topic.
- The intention of this section is to assess students across the breadth of the syllabus in depth. The range of syllabus topics tested in this section may be narrower than that tested in section A.

Question type

- Questions require extended responses.
- Individual questions will develop a single theme.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on sustained reasoning.

Paper 3

Duration: 1 hour

Weighting: 20%

- This paper consists of two compulsory extended-response problem-solving questions.
- A GDC is required for this paper, but not every question part will necessarily require its use.

Syllabus coverage

- Where possible, the first part of each question will be on syllabus content leading to the problem-solving context. Therefore, knowledge of all syllabus topics is required for this paper.

Mark allocation

- This paper is worth **55** marks, representing **20%** of the final mark.
- Questions may be unequal in terms of length and level of difficulty. Therefore, each question may not be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of each question.

Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions will develop from a single theme where the emphasis is on problem solving leading to a generalization or the interpretation of a context.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline in difficulty, from relatively easy at the start of a question to relatively difficult tasks at the end of the question. The emphasis is on problem solving.

Internal assessment criteria—SL and HL

The exploration is internally assessed by the teacher and externally moderated by the IB using assessment criteria that relate to the objectives for mathematics.

Each exploration is assessed against the following five criteria. The final mark for each exploration is the sum of the scores for each criterion. The maximum possible final mark is 20.

Students will not receive a grade for their mathematics course if they have not submitted an exploration.

Criterion A	Presentation
Criterion B	Mathematical communication
Criterion C	Personal engagement
Criterion D	Reflection
Criterion E	Use of mathematics

Criterion A: Presentation

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence or some organization.
2	The exploration has some coherence and shows some organization.
3	The exploration is coherent and well organized.
4	The exploration is coherent, well organized, and concise.

The "presentation" criterion assesses the organization and coherence of the exploration.

A **coherent** exploration is logically developed, easy to follow and meets its aim. This refers to the overall structure or framework, including introduction, body, conclusion and how well the different parts link to each other.

A **well-organized** exploration includes an introduction, describes the aim of the exploration and has a conclusion. Relevant graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document. Appendices should be used to include information on large data sets, additional graphs, diagrams and tables.

A **concise** exploration does not show irrelevant or unnecessary repetitive calculations, graphs or descriptions.

The use of technology is not required but encouraged where appropriate. However, the use of analytic approaches rather than technological ones does not necessarily mean lack of conciseness, and should not be penalized. This does not mean that repetitive calculations are condoned.

Criterion B: Mathematical communication

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration contains some relevant mathematical communication which is partially appropriate.
2	The exploration contains some relevant appropriate mathematical communication.
3	The mathematical communication is relevant, appropriate and is mostly consistent.
4	The mathematical communication is relevant, appropriate and consistent throughout.

The "mathematical communication" criterion assesses to what extent the student has:

- used appropriate mathematical language (**notation, symbols, terminology**). Calculator and computer notation is acceptable only if it is software generated. Otherwise it is expected that students use appropriate mathematical notation in their work
- defined **key terms** and variables, where required
- used **multiple forms of mathematical representation**, such as formulae, diagrams, tables, charts, graphs and models, where appropriate
- used a **deductive method** and set out proofs logically where appropriate

Examples of level 1 can include graphs not being labelled, consistent use of computer notation with no other forms of correct mathematical communication.

Level 4 can be achieved by using only one form of mathematical representation as long as this is appropriate to the topic being explored. For level 4, any *minor* errors that do not impair clear communication should not be penalizsed.

Criterion C: Personal engagement

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of some personal engagement.
2	There is evidence of significant personal engagement.
3	There is evidence of outstanding personal engagement.

The "personal engagement" criterion assesses the extent to which the student engages with the topic by exploring the mathematics and making it their own. It is not a measure of effort.

Personal engagement may be recognized in different ways. These include thinking independently or creatively, presenting mathematical ideas in their own way, exploring the topic from different perspectives, making and testing predictions. Further (but not exhaustive) examples of personal engagement at different levels are given in the teacher support material (TSM).

There must be evidence of personal engagement demonstrated in the student's work. It is not sufficient that a teacher comments that a student was highly engaged.

Textbook style explorations or reproduction of readily available mathematics without the candidate's own perspective are unlikely to achieve the higher levels.

Significant: The student demonstrates authentic personal engagement in the exploration on a few occasions and it is evident that these drive the exploration forward and help the reader to better understand the writer's intentions.

Outstanding: The student demonstrates authentic personal engagement in the exploration in numerous instances and they are of a high quality. It is evident that these drive the exploration forward in a creative way. It leaves the impression that the student has developed, through their approach, a complete understanding of the context of the exploration topic and the reader better understands the writer's intentions.

Criterion D: Reflection

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection.

The "reflection" criterion assesses how the student reviews, analyses and evaluates the exploration. Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.

Simply describing results represents **limited reflection**. Further consideration is required to achieve the higher levels.

Some ways of showing **meaningful reflection** are: linking to the aims of the exploration, commenting on what they have learned, considering some limitation or comparing different mathematical approaches.

Critical reflection is reflection that is crucial, deciding or deeply insightful. It will often develop the exploration by addressing the mathematical results and their impact on the student's understanding of the topic. Some ways of showing critical reflection are: considering what next, discussing implications of results, discussing strengths and weaknesses of approaches, and considering different perspectives.

Substantial evidence means that the critical reflection is present throughout the exploration. If it appears at the end of the exploration it must be of high quality and demonstrate how it developed the exploration in order to achieve a level 3.

Further (but not exhaustive) examples of reflection at different levels are given in the teacher support material (TSM).

Criterion E: Use of mathematics—SL

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used.
2	Some relevant mathematics is used. Limited understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. Limited understanding is demonstrated.

Achievement level	Descriptor
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is mostly correct. Good knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Thorough knowledge and understanding are demonstrated.

The "Use of mathematics" SL criterion assesses to what extent students use mathematics that is **relevant** to the exploration.

Relevant refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

Students are expected to produce work that is **commensurate with the level** of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, or at a similar level.

A key word in the descriptor is **demonstrated**. The command term demonstrate means "to make clear by reasoning or evidence, illustrating with examples or practical application". Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be **thorough** it must be demonstrated throughout.

The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.

Students are encouraged to use technology to obtain results where appropriate, but **understanding must be demonstrated** in order for the student to achieve higher than level 1, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.

Criterion E: Use of mathematics—HL

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used. Limited understanding is demonstrated.
2	Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Some knowledge and understanding are demonstrated.

Achievement level	Descriptor
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and demonstrates sophistication or rigour. Thorough knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is precise and demonstrates sophistication and rigour. Thorough knowledge and understanding are demonstrated.

The "Use of mathematics" HL criterion assesses to what extent students use **relevant** mathematics in the exploration.

Students are expected to produce work that is **commensurate with the level** of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, at a similar level or slightly beyond. However, mathematics of a level slightly beyond the syllabus is **not** required to achieve the highest levels.

A key word in the descriptor is **demonstrated**. The command term demonstrate means to make clear by reasoning or evidence, illustrating with examples or practical application. Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be **thorough** it must be demonstrated throughout. Lines of reasoning must be shown to justify steps in the mathematical development of the exploration.

Relevant refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome. **Precise** mathematics is error-free and uses an appropriate level of accuracy at all times.

Sophistication: To be considered as sophisticated the mathematics used should be commensurate with the HL syllabus or, if contained in the SL syllabus, the mathematics has been used in a complex way that is beyond what could reasonably be expected of an SL student. Sophistication in mathematics may include understanding and using challenging mathematical concepts, looking at a problem from different perspectives and seeing underlying structures to link different areas of mathematics.

Rigour involves clarity of logic and language when making mathematical arguments and calculations. Mathematical claims relevant to the development of the exploration must be justified or proven.

Students are encouraged to use technology to obtain results where appropriate, but **understanding must be demonstrated** in order for the student to achieve level 1 or higher, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.